Worksheet 6

1. Let $A = \{1,2,3\}$ and $B = \{7,2\}$, list all the elements in $A \times B$

What is $|A|$, $|B|$, and $|A \times B|$

2. Prove $A \times (B \cup C) = (A \times B) \cup (A \times C)$

3. Consider following statement, explain if it is true or give a counterexample if it is false.

   Let A, B be sets

   a. $|A \times B| = |B \times A|$

   b. $A \times B = B \times A$

   c. $A \times B \neq B \times A$

4. List the power set for the following sets

   a. $A = \{1,2,3\}$

   b. $B = \{1, \{2,3\}\}$

   c. $C = \{\{1,2,3\}\}$

   d. $D = \{\emptyset, 4\}$

5. How many elements are in each power set for the previous problem

   e. $|\mathcal{P}(A)| =$

   f. $|\mathcal{P}(B)| =$

   g. $|\mathcal{P}(C)| =$

   h. $|\mathcal{P}(D)| =$

   i. What conjecture can you make about the number of elements in power set?

j. Prove the power set of an n-element set contains 2^n elements

(That is to prove: if $|X| = n$, then $|\mathcal{P}(X)| = 2^n$)

6. For each $n \in \mathbb{N}$, let $A_n = (n, \infty)$

   a. $A_2 =$

   b. $\bigcup_{n\in\mathbb{N}} A_n =$

   c. $\bigcap_{n\in\mathbb{N}} A_n =$

7. For each $\alpha \in \mathbb{Z}$, let $A_\alpha = \mathbb{Z}\backslash\{-\alpha, \alpha\}$

   a. $A_2 =$

   b. $\bigcup_{\alpha\in\mathbb{Z}} A_\alpha =$

   c. $\bigcap_{\alpha\in\mathbb{Z}} A_\alpha =$

8. For each $\alpha \in [0,2\pi]$, let $A_\alpha = \{(cos\alpha, sin\alpha)\}$

   a. $A_{\frac{\pi}{6}} =$

   b. $\bigcup_{\alpha\in[0,2\pi]} A_\alpha =$

9. Let $A = \{1,2,3\}$, write each statement in English and then determine whether it is true or false

   a. $\exists x \in \mathcal{P}(A), |x \cap A| = 1$

   b. $\forall x \in A, x^2 - 1 > 4$

   c. $\forall x \in \mathcal{P}(A), A \backslash x \subseteq A$